MiniWorkshop "Calculus of Variations and Partial Differential Equations"

Dipartimento di Matematica e Applicazioni "Renato Caccioppoli"

Università degli Studi di Napoli Federico II

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Roberto Alicandro
University of Cassino

Linear elasticity as Γ-limit of multiwell energies

Abstract

Linear elasticity can be rigorously derived from finite elasticity under the assumption of small loadings in terms of Γ-convergence. This was first done in the case of one-well energies with superquadratic growth and later generalized to different settings, in particular to the case of multiwell energies where the distance between the wells is very small (comparable to the size of the load). I will consider the case when the distance between the wells is independent of the size of the load. In this context I will show that linear elasticity can be derived by adding to the multiwell energy a singular higher order term which penalizes jumps from one well to another. Finally I will derive linear elasticity from a two-well discrete model, showing that the role of the singular perturbation term is played in this setting by interactions beyond nearest neighbours.

Pierre Bousquet
University of Toulouse

Approximation of Sobolev maps with values into manifolds

Abstract

A minimizing harmonic map u from the unit ball $B \subset \mathbb{R}^m$ with values into a manifold N is smooth outside a small singular set. Some of these singularitites are topologically non trivial, in the sense that u cannot be approximated in a neighborhood of this singular set by smooth maps with values into N. In this talk, we present classical and more recent counterexamples and positive results on the density of smooth maps $C^\infty(\overline{B}; N)$ in Sobolev spaces $W^{1,p}(B; N)$, both for the strong and the weak topology. We consider the case when the target manifold N is closed but not necessarily compact, where both the geometry and the topology of N play a crucial role. This is a joint work with Augusto Ponce and Jean Van Schaftingen.

Lorenzo Brasco
University of Ferrara

Regularity issues for the fractional p-Laplacian

Abstract

In this talk we will present some recent results on the regularity of solutions to fractional quasilinear elliptic equations. We will focus on continuity, higher differentiability and logarithmic-type estimates. Some of the results presented are contained in joint works with Eleonora Cinti, Erik Lindgren, Enea Parini and Armin Schikorra.

Matteo Focardi
University of Firenze

The measure and the structure of the free boundary in the lower dimensional obstacle problem

Abstract

In this talk I present the main results of a recent paper in collaboration with E. Spadaro (University of Leipzig) on the regularity of the free boundary for a class of lower dimensional obstacle problems, including the classical scalar Signorini problem. We prove the first results concerning the global structure of the free boundary, in particular showing its local finiteness and its rectifiability.

Maria Stella Gelli
University of Pisa

Relaxation and convexity issues for L^∞-functionals

Abstract

Under general hypotheses on a supremal functional F depending on the gradient we prove the level convexity of its relaxation with respect to the uniform/weak*/weak*-sequential topology. Note that without any coercivity and growth conditions the abstract relaxed functionals above may differ a lot. As a consequence we prove that such a gap does not happen for supremal L^∞-functionals with respect to the weak*/weak*-sequential topology. A key ingredient is to write sublevel sets of the relaxation in terms of those of suitable (envelopes of) different quotients. These results are obtained in collaboration with F. Prinari.

Carlo Mantegazza
University of Napoli Federico II

Evolution of networks in the plane

Abstract

We will present the state-of-the-art of the problem of the motion by curvature of a network of curves in the plane, discussing existence, uniqueness, singularity formation and asymptotic behavior of the flow.

Maria Giovanna Mora
University of Pavia

The equilibrium measure for a nonlocal dislocation energy

Abstract

In this talk I will discuss the minimization problem for a nonlocal energy describing the interaction of positive dislocations in the plane. The interaction kernel is the sum of the Coulomb potential and of an anisotropic term, which makes the potential non-radially symmetric. In the absence of anisotropy the problem corresponds to purely logarithmic interactions and has been studied in a variety of contexts (Ginzburg-Landau vortices, Coulomb gases, random matrices, Ferkel sets); in this case it is well known that the equilibrium measure is given by the celebrated circle law. I will show that the presence of the anisotropy in the kernel changes dramatically the nature of the equilibrium measure, which turns out to be supported on the vertical axis and distributed according to Wigner's semi-circle law. This result is one of the few examples where the minimizer of a nonlocal energy is explicitly computed and the first one in the case of anisotropic kernels. Moreover, it gives a positive answer to the conjecture that positive dislocations tend to arrange themselves in vertical walls.

Giovanni Pisante
University of Campania Luigi Vanvitelli

Duality approach for some variational problems involving polyconvex integrands

Abstract

Duality methods have beed proved to be a very useful tool in studying optimization problems. The theory is however fully developed only in the convex setting. Several attempts have been made to try to generalize the method to non-convex problems. Aim of the talk is to present an approach that has been recently proposed to identify dual problems for a class of polyconvex integral functionals and to discuss how this theory could be applied to recover informations on the minimizers.

Francesco Solombrino
University of Napoli Federico II

Global and local minimizers in prestrained elastic bistrips

Abstract

We study the stable configurations of a thin three-dimensional weakly prestrained rod subject to a terminal load as the thickness of the section vanishes. By Γ-convergence we derive a one-dimensional limit theory and show that isolated local minimizers of the limit model can be approached by local minimizers of the three-dimensional model. In the case of isotropic materials and for two-layers prestrained three-dimensional models the limit energy further simplifies to that of a Kirchhoff rod-model of an intrinsically curved beam. In this case we study the limit theory and investigate global and/or local stability of straight and helical configurations. We also show, by means of a bifurcation analysis, an exchange of stability between the

straight configuration and a branch of local minimizers with so-called hemihelical shape, confirming experimental results. Joint with M.Cicalese and M. Ruf.